Online functional testing with rapid prototypes: a novel empirical similarity method

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Abstract

Functional testing with rapid prototypes is confined to certain areas due to a number of issues: the lack of a reliable similarity method that can solve distorted similarity problems; limited material choices; range of prototype sizes; and distinct material structures between prototypes and actual products. Methods are thus needed to expand the application of functional testing with rapid prototypes, and thus potentially impact the performance and cycle times of current product development processes. In this context, an improved similarity method that utilizes a geometrically simple specimen pair is developed in this paper. A realistic numerical simulation and an experimental mold design example (using a selective laser sintering prototype), demonstrate the validity and impact of the new method.

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Introduction

The performance of product development processes can be sharply improved if one can utilize rapid prototypes as reliable replacements of test products. The utilization of rapid prototypes in place of test products can save significant fabrication effort, especially when the geometry of the product is complex. However, there exist several barriers that prevent the precise prediction of product behavior through testing prototypes.

The traditional similarity method, which is based on the Buckingham Π theorem, is commonly applied to correlate the behavior of scaled physical models[1] and products (Baker *et al.*, 1991; Kline, 1965). For accurate prediction of product behavior through the method, all corresponding dimensionless parameters of the two systems should be identical. When this requirement is satisfied, the prototype is deemed a well-scaled model of the product; otherwise, the prototype is a distorted or distortedly-scaled model. In order to design a well-scaled model, influential system parameters should be chosen, and parameter values should be known. Occasionally, parameters are over- or under-defined, and some parameter values are not known, as the system becomes complex. When reliable parameter values are not available, experiments should be conducted to procure reliable parameter values. Moreover, it is not always possible to design well-scaled models, as dimensionless parameters are coupled functions of physical parameters which are related to base materials and loading conditions (Baker *et al.*, 1991). When a rapid prototyping technique is utilized to construct scaled models, it becomes more difficult to construct well-scaled models due to the limited material choices and unique fabrication schemes of these technologies. The difficulty

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in applying desired loading conditions is another technical problem in realizing well-scaled models.

As an alternative to the traditional similarity method, our novel empirical similarity method aims to circumvent the strict conditions on system parameters, i.e., the identity of corresponding dimensionless parameters of the prototype and the product. We claim that the traditional method is not appropriate to solve distorted similarity problems, as it relies only on restricted system information, the dimensional vectors of two systems. Observing that fabrication effort is heavily dependent on the geometrical complexity of parts, the involvement of a geometrically simple specimen pair is proposed as a way to overcome the limitations from utilizing only dimensional information. One specimen with simple geometric features is fabricated from a rapid prototyping process (prototype specimen), and another from the actual production process (product specimen). Our method measures the state vectors of the specimen pair and the scaled rapid prototype, and the state transformation between specimens is derived. Assuming that the derived state transformation can be also applied to systems with distinct geometrical features, the product states are predicted through an empirically derived transformation. In this paper, the state transformation is assumed to be linear and consistent.

For verification of this concept, simple numerical simulations are performed with ANSYS™. An experiment using aluminum and nylon parts is also performed to demonstrate the utility of the new similarity method. The nylon parts were fabricated by selective laser sintering (SLS). The examples show the superiority of the novel empirical similarity method over the traditional method.

Rapid prototypes for effective functional tests

Industries develop and apply rapid prototyping techniques for two reasons:

(1) To improve design processes by providing timely information; and
(2) To rapidly produce products or tools.

This paper is concerned with the first case, emphasizing the development of functional information. Industries have improved product quality or reduced time to market by utilizing rapid prototypes as geometric models in order to effectively derive customer needs, enhance communication, and detect design faults early (Jacobs, 1992). Some functional tests with rapid prototypes have been performed (Schmidt, 1991), but only in limited areas. Various research efforts are focused on the fabrication of parts with improved functional features that are suitable for functional tests. However, studies that seek to improve test results and increase applicable domains with available rapid prototypes are rare.

Physical prototyping

Industries utilize both virtual and physical models to procure proper information, despite the fact that the applicable problem domains and the accuracy of virtual models are increasing. The trend in industry is to verify product quality and adjust design parameters through physical tests. The test/parameter tuning cycle is repeated until acceptable quality is obtained within allowed cost and time constraints. As this iterative quality improvement cycle is costly and time consuming, in many cases testing of scaled models is performed instead of direct product testing.

In some cases, more expensive scaled-up models are prepared for functional tests. However, it is more common to geometrically scale down, to change materials, and/or to simplify models (Fay, 1993), in order to reduce the physical modeling effort. Wall *et al.* (1991) investigated and evaluated several prototyping techniques (stereolithography, rubber molding, and CNC machining), with duct and bracket samples (Wall *et al.*, 1991). According to their study, the rapid prototyping process usually requires the least cost and time for fabrication of a single part, but available rapid prototyping materials are too limited to perform diverse functional tests.

As shown in Table I, most commercial rapid prototyping systems fabricate prototypes from polymer materials (Aubin, 1994). A metallic prototype, which may be more suitable for functional testing, can be obtained through the conversion of a polymeric prototype to cast metal, and some processes have the capability to directly fabricate metallic parts (Das *et al.*, 1997; Prinz, 1994). However, it may be some time before a commercial rapid prototyping system that can directly fabricate reliable metallic prototypes is realized, due to the high melting point of metals. It is interesting to note that some

Table I Materials available for the direct rapid prototyping

Company	Base materials
3D Systems Inc.	Liquid photopolymer (epoxy resin)
Helisys Inc.	Paper coated with polyethylene
Soligen Inc.	Ceramic powder with liquid blender
Strasys Inc.	Investment casting wax, polyolefin, polyamide
DTM	Nylon, polycarbonate, ProtoFrom composite, TruForm PM, polymer coated sand and bronze etc
Laser 3D	Photopolymers
C-MET	Hard polymer, rubber
D-MET Ltd	Urethan acrylate

companies are developing base materials (e.g. ProtoForm of DTM), and systems (e.g. Actua of 3D Systems), for efficient fabrication of prototypes with focused features (e.g. highly accurate geometry with limited strength). Considering the system price, part accuracy, and fabrication effort, the utilization of metallic rapid prototypes may not be suitable for scaled testing.

Advances in rapid prototyping techniques

The fabrication of electrical interconnects and *in situ* sensors (Beck *et al.*, 1992; Safari *et al.*, 1997; Sun *et al.*, 1997), is a noteworthy research area for functional testing with rapid prototypes. Rapid prototypes with electrical interconnects and *in situ* sensors open a new avenue for functional tests by enhancing the monitoring of the internal system behavior. The application of the embedded sensors (Halwel *et al.*, 1991; Smith *et al.*, 1992) shows the possibility of these techniques. Recently, fabrication of rapid prototypes with multiple and functionally gradient materials (Fessler *et al.*, 1997; Griffith *et al.*, 1997; Jepson *et al.*, 1997), has emerged as a new research area. Rapid prototyping with functionally gradient materials may enable the fabrication of prototypes with manufacturing process dependent characteristics (e.g. surface hardening of parts after annealing). The research effort has the potential to enable functional tests with rapid prototypes to predict the functional behavior of products with non-homogeneous material structures.

Along with these relatively new studies, research to fabricate geometrically accurate parts with desired functional attributes (e.g. strength, water resistance, heat resistance), has been continuously carried out. Because of these efforts, the application of rapid prototypes as functional prototypes is closer to realization. However, functional tests with rapid prototypes may produce erroneous test results due to the following reasons:

- Most rapid prototyping processes, such as stereolithography, selective laser sintering, shape deposition, and 3D printing, fabricate parts by continuously generating contoured layers from a sheet, liquid, or powder, and combining the layers (Jacobs, 1992). Due to the layer-additive or subtractive fabrication schemes, most rapid prototypes show orthotropic material structures to some degree (Nelson *et al.*, 1993). In extreme cases, rapid prototypes show delamination of layers.
- In spite of significant effort to expand base materials, polymers are still the most popular and dependable materials. Polymers show material behavior which is distinct from that of metals in many aspects. Non-linear stress-strain curves and highly temperature-dependent material properties are typical examples (Birley *et al.*, 1988).

Contrary to expectations, very little literature exists on the subject of functional testing with rapid prototypes (Dornfeld, 1995; Steinchen *et al.*, 1995). Most such studies are based on the traditional similarity method, and most just experimentally examine the test results without providing a way to improve test results.

Novel empirical similarity method

Given a prototype and a target system, if any of their corresponding dimensionless parameters are not identical, the two systems are said to be distorted. Such distortion is frequently encountered even when scaled models are fabricated from common prototyping techniques. However, the problems become more severe when rapid prototypes are used as scaled models. For instance, the freedom to design well-scaled models is decreased due to the limited material choices. The problems mentioned in the section "physical prototyping" make similarity-based design more difficult. In this section, the inherent limitations of the traditional similarity method are clarified, and our new similarity method to overcome these limitations is introduced.

Concept of the empirical similarity method

The traditional similarity method presumes that the state of interest of the product and the scaled model systems, q_p and q_s, can be represented with following equations:

$$q_P = f(a_1, a_2, \cdots, a_n)$$
$$q_S = f(b_1, b_2, \cdots, b_n) \tag{1}$$

where f is an unknown function, and a_1 and b_1 are corresponding system parameters. If all the corresponding dimensionless parameters π of the two systems are identical, i.e.

$$(\pi_P)_i = a_1^{r_1} \cdot a_2^{r_2} \cdots a_n^{r_n} = b_1^{r_1} \cdot b_2^{r_2} \cdots b_n^{r_n} = (\pi_S)_i\ , \tag{2}$$

then the relationship between the state variables becomes

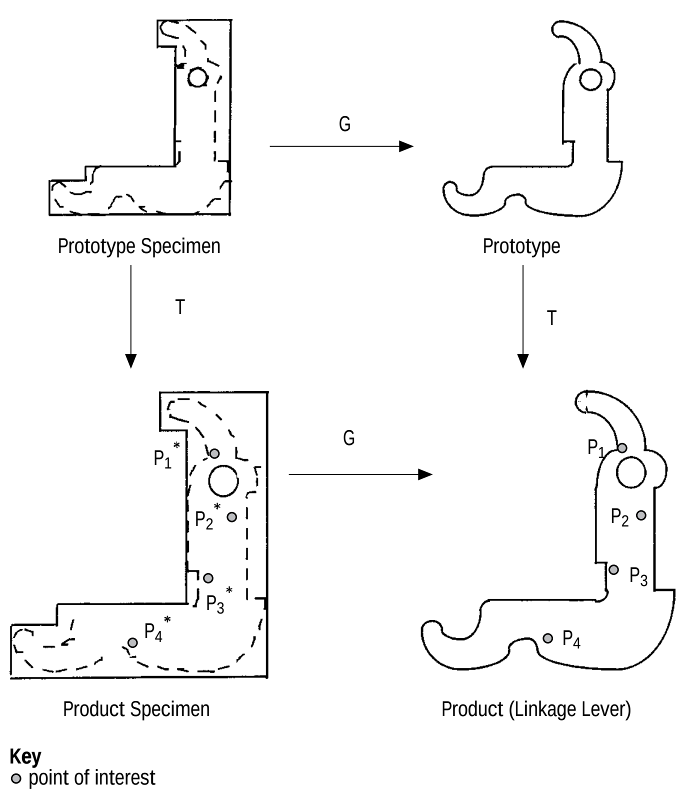
$$q_P = \lambda_q \cdot q_S\ . \tag{3}$$

In equation (2), r_1 is a rational number, and the scale factor of the state, λ_q, in equation (3), is a constant which is a function of given parameters of the product and the prototype. The equality of the corresponding dimensionless parameters, equation (2), can be considered as constraints to the design of well-scaled prototypes. If either the assumption of the functional form, equation (1), or constraints on the prototype, equation (2), are violated, the two systems are distorted. As a consequence, the state of the product cannot be derived accurately from the prediction equation (3).

Several Π theorem-based studies have been carried out to solve distorted similarity problems (Bazant, 1994; Murphy, 1971). Among them, Murphy's work is notable. Murphy proposed a strategy to relax the similarity constraints by intentionally casting the prediction equation into the more complex form, in order to design the prototype with more freedom. In parallel to the dimensional analysis based studies, the symmetry method can be utilized to solve similarity problems. The symmetry method finds solutions of nonlinear differential equations by transforming the solutions of simple linear differential equations. For boundary value problems, the symmetry method can be considered as a generalization of dimensional analysis (Bluman and Kumei, 1989). However, both Murphy's work and the symmetry method assume that one already knows the governing equations.

As the name implies, the novel empirical similarity method empirically derives the relationship between the state vectors of the prototype and the product from a specimen pair. In comparison to the traditional method, the new method relates two vectors instead of the scalar relation shown in equation (3), so that the relation is not restricted to a stretching transformation. Thus, one can represent more complex relationships between states. In comparison to the traditional similarity method, our new method neither presumes the identity of the governing equation nor the design constraints on the prototype. Also, one need not know the exact system parameters of the prototype and the product, as the relationship between states is derived not from knowledge of relevant system parameters but from empirically measured states. Potentially, our new method can predict the functional behavior of products from prototypes, provided geometrical similarity between the two systems is maintained. Figure 1 illustrates the

Figure 1 State transformation between prototype, target and specimen pairs (**G** denotes the geometric transformation and **T** denotes the similarity transformation)



basic concept of the new method, showing the possible geometry and loading conditions, and corresponding measurement points of the specimens. The prototype is considered as a physical entity whose geometric features are the same as those of the product. Similarly, the product specimen is considered as an entity whose non-geometric features (material properties and loading conditions), are the same as those of the product, but whose geometry is distinct from the product. The prototype specimen has no direct common features with the product. However, it has common geometric features with the prototype and common non-geometric features with the product specimen. In this study, the product and the product specimens are fabricated with the same production process (e.g. machining, casting, pressing etc.), and the scaled prototype and the prototype specimens from a rapid prototyping process. From the measured state vectors of the specimen pair (the product and the prototype specimens), the state transformation due entirely to variation of non-geometric features can be extracted. If the variation of non-geometric features is consistent, one can predict the states of the product from the extracted transformation and the states of the prototype.

Similarity and geometrical transformation matrix

The geometrical state transformation **G**, between the prototype specimen and the prototype, and possibly the product specimen and the product, represents state variations due to pure geometrical variations. Similarly, the similarity state transformation **T** represents state variations due to the variation of non-geometric features when the geometrical features are maintained. As an initial approach, **T** is assumed to be linear. As **T** is a scaled identity matrix for well-scaled problems, the transformation matrix **T** with a maximal diagonal norm is sought. The diagonal norm of a matrix **T** is defined as

$$\|\mathbf{T}\| = \sum_{i=1}^{N} (t_{ii})^2 , \tag{4}$$

where N is the size of the considered state vector and the t_{ii} is a diagonal term of **T**. The procedure to derive the linear state transformation matrix is as follows:

(1) Find the initial transformation matrix $\mathbf{T}_0$ defined as

$$\mathbf{T}_0 \cong \left(\frac{1}{N} \cdot \sum_{i=1}^{N} \frac{X_i^{PS}}{X_i^{RS}} \right) \cdot \mathbf{I}_N , \tag{5}$$

where X_i^{PS} is the state of the product specimen at the local point p_i, and X_i^{PS} is the state of the scaled rapid prototype at the corresponding local point p_i*, as shown in Figure 1. Also, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. The matrix $\mathbf{T}_0$ can be considered as the state transformation that assumes the two systems are well-scaled.

(2) Let the complete state transformation **T** be a composition of $\mathbf{T}_0$ and a matrix $\delta\mathbf{T}$,

$$\mathbf{T} = \mathbf{T}_0 + \delta\mathbf{T} . \tag{6}$$

The matrix $\delta\mathbf{T}$ represents the state transformation that compensates for the pure dissimilarity between the two systems. **T** is derived by perturbing the initial matrix $\mathbf{T}_0$, maximizing the diagonal norm $\|\mathbf{T}\|$

(3) Find the unknown compensation matrix $\delta\mathbf{T}$ from

$$\delta\mathbf{T} = (\mathbf{X}_{PS} - \mathbf{T}_0 \cdot \mathbf{X}_{RS}) \cdot \mathbf{X}_{RS}^{+} , \tag{7}$$

where X_{PS} and X_{RS} are the state vectors of the product specimen and the scaled rapid prototype specimen, respectively. Here, X_{RS}^{+} is the Moore-Penrose pseudo-inverse matrix (Strang, 1988), of X_{RS}^{+} that satisfies

$$\mathbf{X}_{RS} \cdot \mathbf{X}_{RS}^{+} \cdot \mathbf{X}_{RS} = \mathbf{X}_{RS}$$
$$\mathbf{X}_{RS}^{+} \cdot \mathbf{X}_{RS} \cdot \mathbf{X}_{RS}^{+} = \mathbf{X}_{RS}^{+} . \tag{8}$$

If $\delta\mathbf{T}$ is negligible, the two systems can be considered well-scaled. A certain norm of $\delta\mathbf{T}$ (e.g. the ratio of the square sum of elements of **T** and $\delta\mathbf{T}$), can be considered as the measure of the distortion between the two systems.

(4) The state vector of the product, $\mathbf{X}_p$, can be predicted from

$$\mathbf{X}_P = (\mathbf{T}_0 + \delta\mathbf{T}) \cdot \mathbf{X}_R , \tag{9}$$

where $\mathbf{X}_R$ is the state vector of the scaled rapid prototype.

Numerical and experimental evaluations

In the following sections, the feasibility of our new method and the prediction accuracy of the concept are numerically and experimentally tested, and the impact of the new method is discussed.

Numerical example – rod under the static load

In the first example, the principal strains of the aluminum rod, shown in Figure 2, are predicted with the traditional and new empirical similarity methods, and the results are compared. In Figure 2, the geometry of the rod, loading conditions, and seven randomly selected points of interest are illustrated. In order to predict the strain of the aluminum rod at the points of interest, the strains of the nylon rod, the aluminum specimen, and the nylon specimen are computed with finite element (FE), simulations using ANSYS™. The geometry, boundary conditions and corresponding recording points of the specimen rods are shown in Figure 3. The geometry of the specimen shown in the figure is one of the possible choices, and one can design the specimen so that any of its interior points are projected to an interior point of target rod. The relevant material properties for the FE simulations are listed in Table II. For clear comparison of two similarity methods, the nylon part made by rapid prototyping is assumed to be highly orthotropic.

Several factors make prediction with the traditional method inaccurate. First, the nylon part is orthotropic, whereas the aluminum part is isotropic. Additionally, the nylon rod shows geometrically nonlinear behavior due to large deformations, the dimensionless Poisson's ratio is not identical, and the same steel shaft is used for both nylon and aluminum rod tests. The similarity results shown in Figure 4 illustrate the improved prediction accuracy of the new method. For

Figure 2 Geometry, boundary conditions and the points of interest of the target and specimen rod for the numerical example: (a) target rod; (b) specimen rod

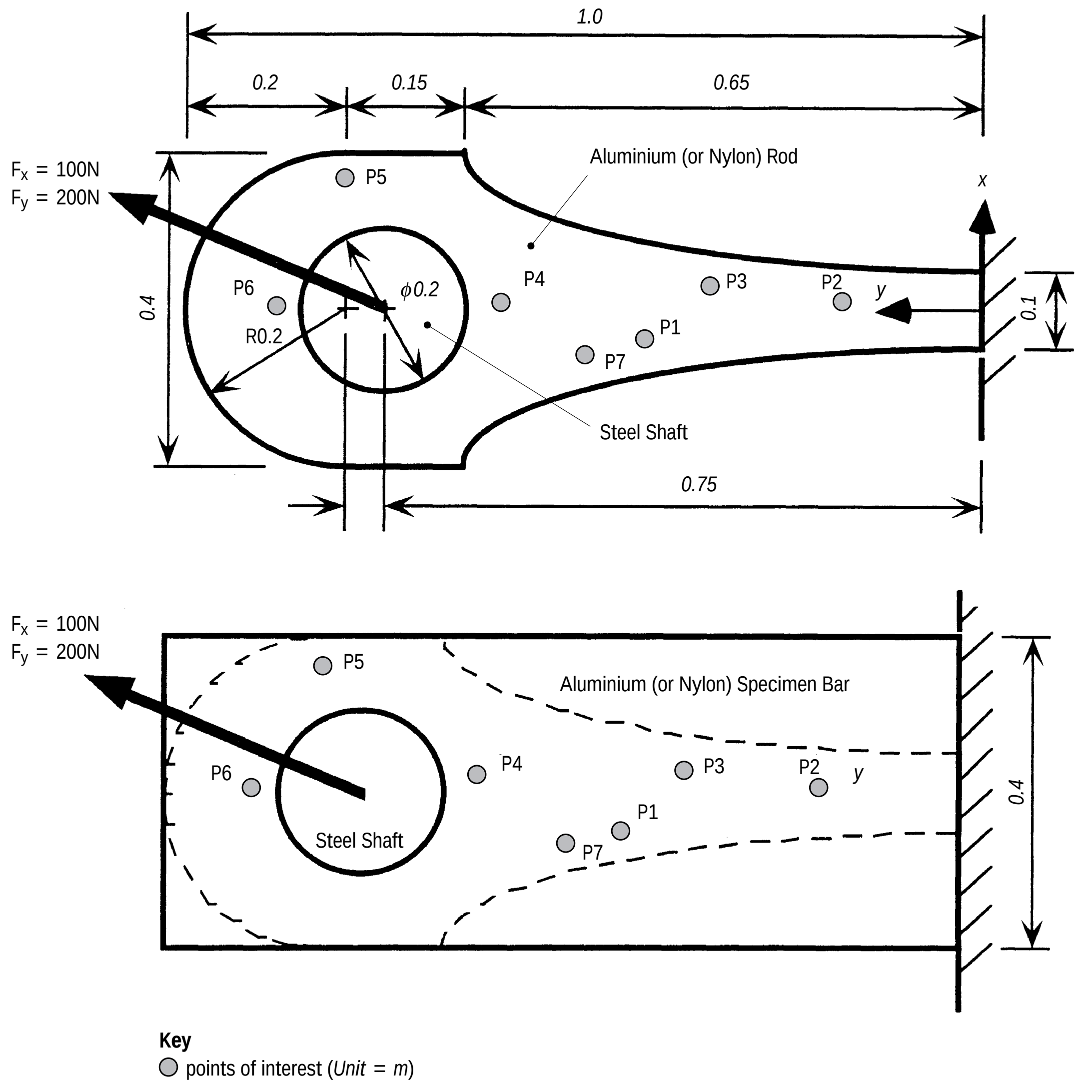
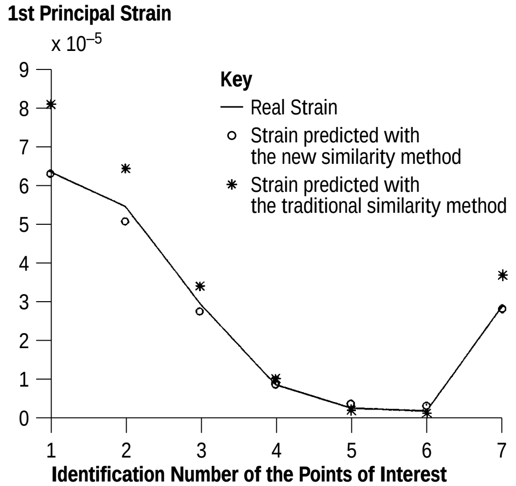


Figure 3 Similarity results of the rod example



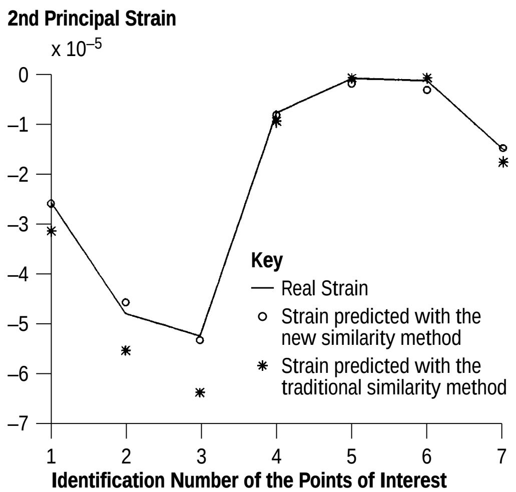
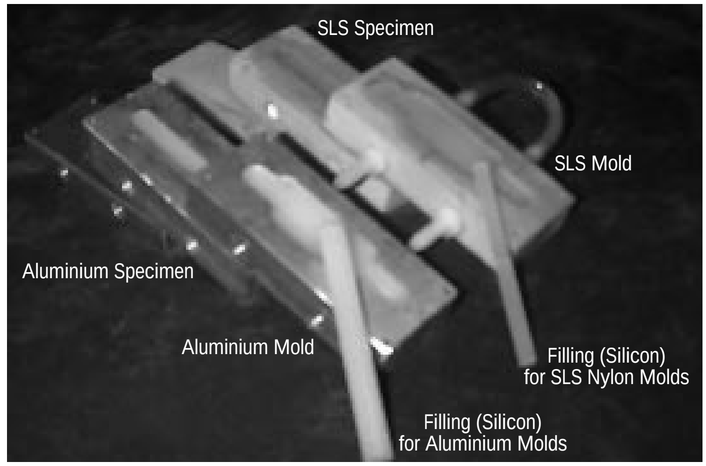


Figure 4 Aluminium and SLS molds for the thermal experiment



the traditional method, the Young's modulus of nylon was assumed to be 12 GN/m^2, the mean value of the x and y directional Young's moduli. In Table III, the prediction error percentages are listed, showing dramatically improved prediction accuracy of the new method compared to the traditional method, except for the second principal strain at P6.

Thermal experiment with SLS and aluminum molds

A thermal model of a mold die is a critical design tool for ensuring the quality of molded parts. In order to improve the productivity of the molding process and the quality of molded parts, several researchers have studied modeling, design and fabrication of the mold die with cooling channels to control heat transfer (Erhun and Advani, 1993; Sachs *et al.*, 1995). However, it is not easy to perform reliable transient thermal simulations due to the complexity of the solidification process. On the other hand, physical modeling, as a substitute or supplement for incomplete virtual modeling, requires significant cost and time, as the geometry of the mold cavity is very complex in many cases.

A desktop experiment that mimics the molding process was conducted to validate scaled transient thermal testing with various rapid prototypes, using SLS-fabricated molds. A photograph of the machined aluminum mold dies and the SLS nylon dies is shown in Figure 5, and the experimental setup is shown in Figure 6. The detailed geometry of the molds with the holes for thermocouples is illustrated in Figure 7. Initially, the SLS nylon and aluminum mold dies were filled with distinct silicones (one for low temperatures, the other for low temperature glue guns), and heated (24 hours for the SLS molds, and 4 hours for the aluminum molds), to maintain uniform initial temperatures (120°C and 200°C, respectively). The molds remained under free convection for two minutes during the experimental setup. After the setup was complete, the cooling water

Table II Material properties of the product and the prototype for the rod example

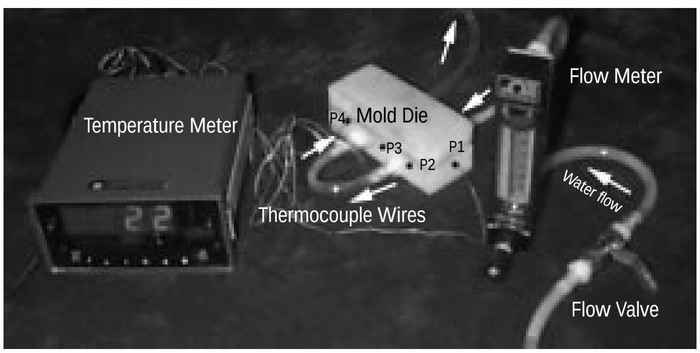
Properties	Aluminium	Nylon	Steel
Young's modulus (GN/m^2)	E = 73	$E_x = 14$, $E_y = 10$	E = 193
Poisson's ratio	V = 0.33	$V_x = 0.27$, $V_y = 0.31$	V = 0.29

Table III Comparison of the strain prediction error percentage

	E% with traditional method	*E*% with new empirical method
ε_1 at		
P1	27.4	0.7
P2	18.2	7.1
P3	15.5	7.5
P4	19.2	0.3
P5	25.6	6.5
P6	39.2	18.8
P7	28.6	2.2
ε_2 at		
P1	22.5	0.6
P2	15.2	4.7
P3	21.2	1.4
P4	22.4	5.6
P5	31.2	15.3
P6	41.2	55.0
P7	17.8	1.9

Note: $E = |\text{Real strain-Predicted strain} \div \text{Real strain}| \times 100\%$

Figure 5 Experimental setup of the mold experimental example



flow valve was opened, and transient temperatures were measured with J-type thermocouples. During the cooling process, the SLS molds showed no permeability to the cooling water. The thermocouple wires were plugged into 4 deep holes (P1 to P4), as shown in Figure 7, in order to measure the temperature of the points close to the cavity boundary. The experiment was repeated six times, and the average values were used for the similarity calculation. As we are interested in the time history of the temperature at a point, the state vector was composed of the temperature measured periodically at a single point of interest. The measurement period was 10s and 100s for aluminum and nylon, respectively. These values were chosen by approximating the solidification time of the mold fill materials. The relevant system parameters are listed in Table IV.

The aluminum and SLS nylon mold dies are mutually distorted mainly due to the boundary conditions. For example, the temperature ratio of the exterior surface and the fill material is distinct in each case. Also, some material properties (e.g. enthalpy of the fill materials), are not known. Thus, it is difficult to predict the temperature history of the aluminum mold with the traditional method.

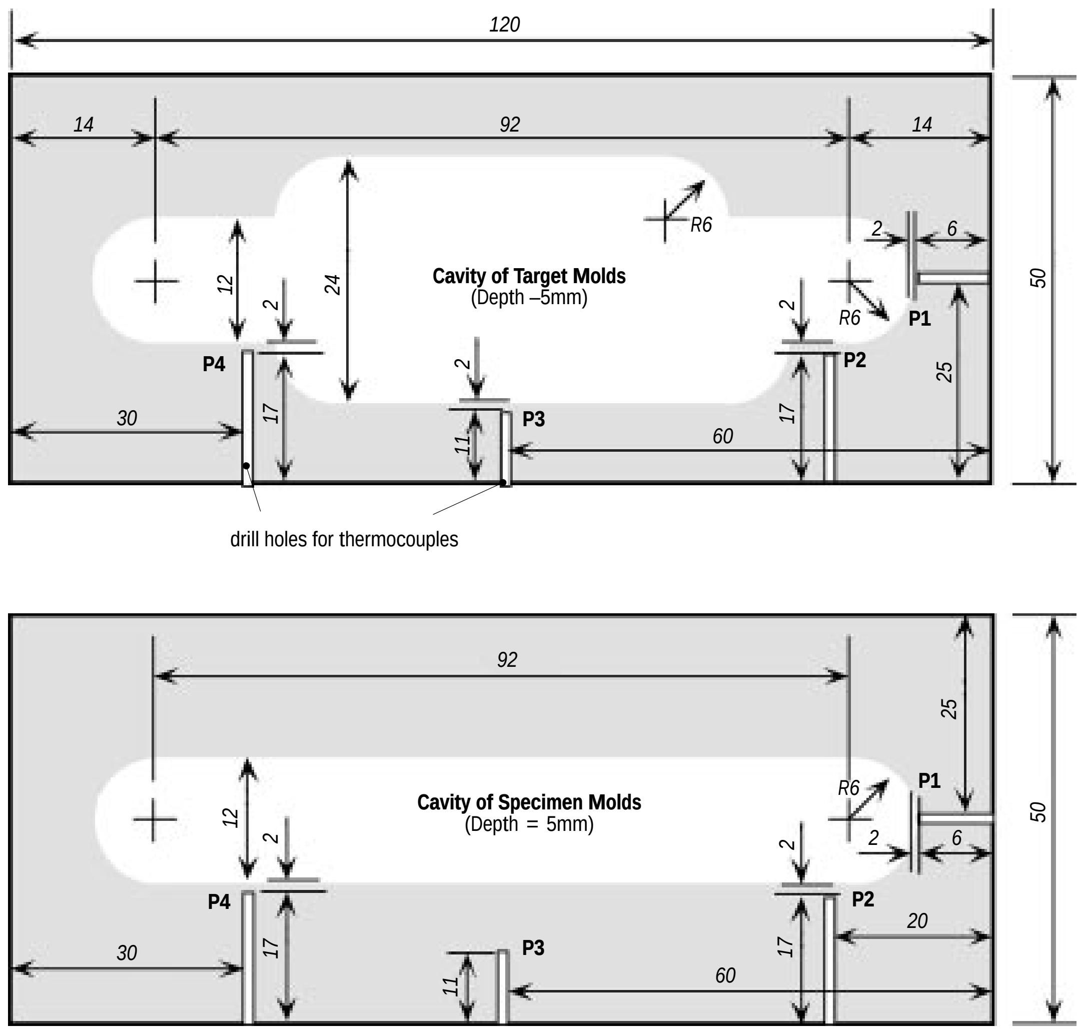
In Figure 7, the similarity results are shown, indicating the potential of the new method to solve distorted nonlinear transient problems. As shown in the figure, the temperature histories of the mold dies are distinct from one measurement point to another, and the trends are nonlinear. At all points, the prediction error during the first half period is less than 3 percent. However, the prediction error during the second half period reaches as high as 15 percent. Possible sources of prediction error include:

(1) non-uniform initial temperature distribution of SLS molds, caused temperature fluctuations in the oven;
(2) fluctuation of the flow rate of the cooling water;
(3) geometrical warpage of the SLS molds; and
(4) imitations in the proposed linear transformation.

Conclusion and future research directions

The novel empirical similarity method presented in this paper can increase both the prediction accuracy and application areas of scaled testing with rapid prototypes. As shown in the numerical and experimental examples, the state prediction accuracy can be improved through the new method. Moreover, the new method can predict the state without knowledge of some system parameters. Because of this, one can perform online functional test without rigorous pre-tests to derive system parameters. As the new method requires extra effort to perform a specimen test, it may be argued that the new method requires more testing effort in comparison to the traditional method. However, this is balanced against the fact that the traditional method also requires pre-tests in order to verify the validity of the test results.

Figure 6 Geometry and temperature measuring points of the target and specimen mold dies for the experimental example: (a) target mold die; (b) specimen mold die

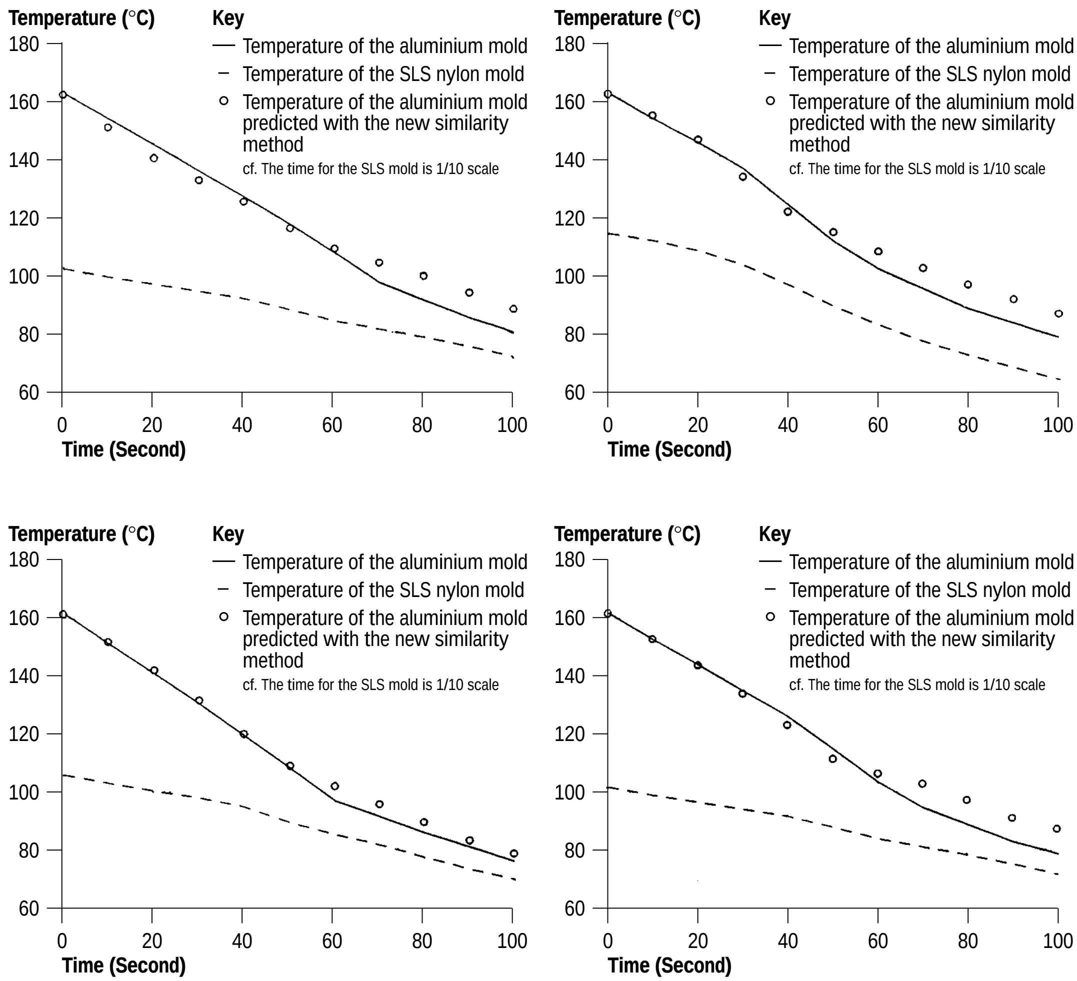


Even though a significant increase in prediction accuracy is demonstrated, the prediction accuracy of the new method still needs improvement to be used confidently in place of direct product tests. Some possible approaches to improving the method include establishing analytical state transformation equations instead of the simple matrix transformation, utilizing multiple specimens recursively, and transforming both the states and independent variables (e.g. local coordinates, time etc.). Other suggested research areas are:

- developing errors measure that can estimate the prediction error bound;
- categorizing the sources of distortion and investigating the characteristics (e.g. eigenvectors of the transformation matrix), of the geometrical and similarity transformation matrices;
- extensive numerical and experimental studies to clarify the limitations and possible improvements of the new method; and
- theoretical proof of the concept through the boundary element or other numerical methods.

Our research parallels efforts to modify or improve rapid prototyping systems for functional testing, and the two research efforts are interdependent. The new empirical similarity method by itself cannot enable correlation between rapid prototypes and products, if rapid prototypes cannot withstand the testing conditions (e.g. permeability of fluid tests), or cannot be fabricated with desired properties (e.g. conductivity for magnetic problems). Likewise, well-scaled models may not be possible to produce even though the number of prototyping materials is increasing. Thus

Figure 7 Similarity results of the mold example: (a) measured and predicted temperature history of the aluminium mold at P1; (b) measured and predicted temperature history of the aluminium mold at P2; (c) measured and predicted temperature history of the aluminium mold at P3; (d) measured and predicted temperature history of the aluminium mold at P4 (the time for the SLS mold is 1/10 scale)



there is a need to integrate advanced rapid prototyping techniques and similarity methods for effective product testing in broad problem domains.

Table IV System parameters of aluminium and SLS mold dies

	Aluminium molds	SLS nylon molds
External surface		
Temperature T_o (°C)	23	23
Convection coefficient	n/a	n/a
Cooling channel		
Temperature T_c (°C)	18	18
Flow rate (liter/sec)	0.13	0.38
Assumed material properties		
Heat conductivity k (W/mK)	177	0.245
Heat capacitance C_p(J/kg K)	875	460
Density ρ (kg/m^3)	2,720	1,140

Note

1 The terms scaled physical model, scaled model and prototype are used interchangeably in this paper.

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